

# On The relation between Superconductivity and Quantum Hall Effect

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## **Abstract**

We introduce a model of superconductivity and discuss its relation to the quantum Hall-effect. This kind of relation is supported by the well known SQUID results. The concept of pure gauge potential as it is involved in various theoretical models concerning solid state effects in magnetic fields is also discussed.

The main properties of superconductors are according to [1] that the magnetic field does not penetrate into the superconductor, i. e.  $B_{in} = 0$  and that no macroscopic volume current can flow in a superconductor. Thus any electric current which flows in a superconductor must be a surface current. These properties refer to a  $2 + 1$ -D theory of superconductivity, where the electromagnetic potentials are pure gauge potentials [2], i. e. with vanishing field strength  $F$ ,  $B \in F$  which should be described by a Chern-Simons-action functional. However, one must keep in mind that  $B_{in} = 0$  or any vanishing of conjugate variables in quantum mechanics have a limit according to the uncertainty relations [3]. Moreover, considering the short range photons in Meissner-effect [1] and the fact that in the Maxwell-Chern-Simons-action in  $2 + 1$ -D the photons become massive and short ranged [4] (see also Ref. [1f]), we are led again to the  $2 + 1$ -D theory of superconductivity. Furthermore, note that the most important hint about the pure gauge character of the electromagnetic potential in superconductivity comes from the coherent phase of the wave function of the whole system of the Cooper-pairs in the BCS-theory [1]. Hence, its phase is given by the line integral of the electromagnetic potential which means that this one is a pure gauge potential which results in the flux quantization. This shows definitely that the main concept of the BCS-theory is the pure gauge potential describing the main object of the theory, i. e. the wave function of Cooper-pairs [5]. Moreover, the possibility of the Cooper-pairs is a quantum mechanical result in  $2 + 1$  dimensions, i. e. in a 2-D potential [1d], [6]. In view of the accepted fundamentality of the BCS-theory this is enough hint about the fundamentality of the mentioned  $2 + 1$ -D point of view which contains also the pure gauge conception.

The usual theory of superconductivity is based on modifications of the Maxwell's electrodynamics according to the phenomenological London's equations, or according to the Ginzburg-Landau-model which are belived to have their microscopic explanation in the BCS-theory. Nevertheless, also the fundamental concept of the Ginzburg-Landau-theory is the 2-dimensional pure gauge potential from which one obtains not only the Ginzburg-Landau-equations but also the Landau's coherence-length [1] [2]. In the same manner also the main concept in London's equations is that of the pure gauge potential which results in the flux quantization [2]. Furthermore, also in case of the Josephson effect which is considered beyond of the flux quantization as a manifestation of the BCS-theory, a new general approach contains the 2-dimensional pure gauge potential [8]. Thus, also the various sub-models and sub-effects of the

superconductivity and weak superconductivity, i. e. superconductivity in lower or higher magnetic fields all demonstrate variations of the pure gauge potential picture. Furthermore, beyond the mentioned methods most of other perturbative techniques in solid state physics like the methods of Green's function or of pseudo-potentials contain also the concept of pure gauge potential *in multiply connected regions*. Close to this concept it is related the concept of multy valued functions and the relation is based on the invariant relations between the topological properties of manifolds and the invariant properties of functions defined on the manifolds, e. g. via Morse-theory, harmonic functions, etc.

On the other hand, consider the fundamental differential geometric discrepancy between the Maxwell's equations  $d^\dagger F = j$  and the London's equations  $F = dj$ . Recall also the discrepancy with the Ginzburg-Landau's equations for higher magnetic fields which are given by  $d^\dagger F = J$ . Furthermore, considering the Ohm's equations  $F = *\rho J$  with  $\rho$  as the resistivity matrix [9], we have again a discrepancy with the Maxwell's equations. Here  $F$  and  $J$  are the field strength and current density two-forms respectively, whereas  $j$  is the current density one-form. Thus, in all effects concerning the conductivity we need additional "phenomenological" relations for currents and conductivity usually called the "material equations" beyond the elaborated Maxwell's equations. Nevertheless, one must keep in mind that  $J$  and  $j$  are referring to two different current densities, namely to the electromagnetic and to the electric ones respectively (see below). The difference is just the electromagnetic potential which appears in these models as a pure gauge potential [2].

These are discrepancies arising from the hierarchy of the 3+1-D consideration of conductivity which refer to a possible solution of discrepancies within a 2+1-D model. Of course, related with these questions are also the hierarchy of the band-theory and phonons in the solid state physics. According to some new field theoretical approaches to quantum Hall-effect (QHE) and superconductivity it seems that the general foundations of these effects does not reflect any band-structure properties [10][11] [5]. Furthermore, there are the so called non-phonon mechanisms models which also explain the high- $T_c$  superconductivity according to the pure electronic behaviour of superconductors [5]. Therefore, one can expect that it should be possible to relate not only the high  $T_c$  superconductivity but the whole superconductivity with QHE for which there are already non-phononic models [10][12]. One advantage of this approach will be the possibility to understand the notion of the critical magnetic induction  $H_c$ . Hence, according to a

possible unified 2 + 1-D approach to QHE and superconductivity, if the exterior magnetic field increases the value of  $B_c$  then the superconductivity of the particle system proceeds into the QHE which should be observable if the system is prepared in the proper way (see below). Hence, there were already hints in this direction according to which the ground state of a system of non-interacting particles with fractional statistics can be considered as a new kind of high- $T_C$  superconductor [11]. Thus, there are enough hints about the necessity of a 2 + 1-D electrodynamical approach to the superconductivity [5].

Recently, we showed that also a microscopic theory of the integer quantum Hall-effect (IQHE) is possible within a Chern-Simons-Schrodinger theory without any relation to the band-theory [12]. Furthermore, there are already models of fractional Hall-effects (FQHE) of the same Chern-Simons-type which are also independent of the band-theory [13]. Thus, there should be beyond the usual solid state theoretical approaches to the conductivity a "parallel" 2 + 1-D gauge field theoretical approach which should incorporate also the Chern-Simons electrodynamical descriptions of the conducting systems. In view of the fact that the usual, i. e. the semi-classical conductivity, the quantum Hall-effects (IQHE and FQHE) and the superconductivity (low and high- $T_c$ ) are all various forms of the electromagnetic conductivity, it seems to be natural to look on a general theory of conductivity connecting all of its features.

Here we introduce a brief theoretical model which incorporates the main electromagnetic aspects of the superconductivity and of the QHE, where we consider both systems as 2 + 1-dimensional systems [12] [13] [14].

To begin recall that the verification of the BCS-theory of superconductivity follows from the flux quantization and the flux quantization is a result of pure gauge field character of the electromagnetic potential [2]. More precisely, the flux quantization is the "quantum mechanical" or global, i. e. invariant expression of the pure gauge character of the electromagnetic potential involved in the electromagnetic current (see below) [2]. This is in accordance with the above mentioned fact that for a superconductor  $B_{in} = 0$ .

It is in view of the global character of the wave function in quantum mechanics that locally vanishing effects which are therefore non-observable in the classical level for example by Lorentz-force equations, become observable in quantum mechanics. Furthermore, the local pure gauge property of an electromagnetic potential is given by the local expression of the vanishing of electromagnetic current density, i.

$$e. J_m^{(em)} = \frac{-ie}{2M_e}(\psi^*(\partial_m\psi) - (\partial_m\psi^*)\psi) - \frac{e^2}{M_e}\psi^*A_m\psi = 0 \text{ for all } \psi \text{ [1], where } M_e \text{ is the mass of electron}$$

and we set  $\hbar = 1$  and  $m, n = 1, 2$  in  $2 + 1$  dimensions. Moreover, one can obtain also the London's equations from the same vanishing of electromagnetic current density.

Now let us observe that, the proper current density for the superconductivity described by the London's equations is the *electric* current density which is valid also in presence of low magnetic fields, i. e. for  $\omega_c \tau \ll 1$ . It is given by  $j_m^{(e)} = \frac{-ie}{2M_e}(\psi^*(\partial_m \psi) - (\partial_m \psi^*)\psi)$ . On the other hand in case of QHE, i. e. in presence of higher magnetic fields or in the limit  $\omega_c \tau \gg 1$ , the *electromagnetic* current density is given by  $J_n^{(em)} = j_n^{(e)} - \frac{e^2}{M_e} \psi^* A_n \psi$ . Thus, if we set, as usual [1] the  $J_n^{(em)} = 0$

$$j_n^{(e)} - \frac{e^2}{M_e} \psi^* A_n \psi = 0 \quad (1)$$

These could be considered as the local solutions of London's equations or equivalently as the London's equations themselves [1]. Thus, the time and exterior spatial derivative of equations (1) result in the original version of London's equations mentioned above. On the other hand, if we consider the  $V_\alpha = J_\alpha^{(em)} = j_\alpha^{(e)} - \frac{e^2}{M_e} \psi^* A_\alpha \psi$  with  $\{\alpha, \beta, \gamma\} = \{1, 2, 3\}$  as a  $2 + 1$ -D vector potential which obey the Lorentz-condition in view of the continuity equation for  $J_\alpha^{(em)}$ , then its dynamics following its pure gauge character according (1) should be given by a Chern-Simons-action:

$$\int \epsilon^{\alpha\beta\gamma} J_\alpha^{(em)} \partial_\beta J_\gamma^{(em)} \quad , \quad (2)$$

Hence, the constraint equations of this action are given by the relation (1).

Thus, the equations of motion and the constraint equations of the action (2) are the London's equations in  $2 + 1$  dimensions  $\epsilon^{\alpha\beta\gamma} F_{\beta\gamma}(V) = \epsilon^{\alpha\beta\gamma} \partial_\beta J_\gamma^{(em)} = 0$ . Therefore, in view of the gauge invariance of the electromagnetic field strength, London's equations are the local gauge invariant expresion of the pure gauge character of the electromagnetic potential.

On the other hand, it is known also that QHE [9] is also related with the pure gauge character of the electromagnetic potential expressed by the Chern-Simons-action [12]. We showed [12] also that the characteristic currents of IQHE, namely its edge currents result also from the pure gauge character of the electromagnetic potential in IQHE according to the constraints of the theory under the typical integer

quantum Hall conditions [15]. Therefore, it seems plausible that the superconductivity and QHE become related with each other in view of the fact that they manifest various properties of the electromagnetic pure gauge potentials.

Recall further that in the language of differential geometry the Londons equations [1] in  $2+1$  dimensions are given by  $dj = (\lambda)^{-1}dA$ , where  $j$ ,  $A$  and  $\lambda$  are current density one-form, the gauge potential one-form and the London's penetrating depth  $\lambda = \frac{M_e}{ne^2}$  respectively, if we set  $\mu_0 = 1$ . On the other hand the Ohm's equations [9] for Hall-effect are given also in  $2+1$ -dimensions by  $J^{(em)} = \sigma dA$ , where  $\sigma$  is the conductivity matrix. Furthermore, in  $2+1$  dimensions we can introduce a geometrical current density  $J = *\lambda dj^{(e)}$ . Hence,  $J$  obeys also the continuity relation  $*\partial * J = 0$ . Thus, we could have a formal relation  $J = *dA$  which is similar to the Ohm's equations of QHE. Recall also that  $J^{(em)}$  and  $j^e$  both have the dimension  $L^{-2}$  in accordance with the particle density on a surface.

It is obvious from such a constellation that the geometrically introduced current density  $J$  can be identified with the current density of the Ohm's equations  $J^{(em)}$  only if the conductivity matrix  $\sigma$  becomes equal to the  $SO(2)$  matrix for the superconducting case which characterizes the rotation [16]. We will show that this condition which is equivalent to  $\sigma_H = 1$  is indeed the case corresponding with the mentioned ground state of the quantum Hall-system [11] which includes also the empirical fact of vanishing of longitudinal conductivity in QHE [15].

The Chern-Simons-action functional of our model for superconductivity which is a slight generalization of the action (2) and from which we can obtain the London's equations as the equations of motion is the following one defined on a  $2+1$ -D manifold  $M = \Sigma \times \mathbf{R}$  with boundary.

$$\int \epsilon_{\alpha\beta\gamma} C_\alpha \partial_\beta C_\gamma \quad , \quad (3)$$

with  $C_\alpha = \lambda j_\alpha^{(e)} - \sigma_H A_\alpha$ , where the  $\sigma_H$  is the locally constant dimensionless parameter called the Hall-conductivity. The  $j_\alpha^{(e)}$  is the usual electric current density of a it non-interacting system of electrons.

If we set the value of the Hall-conductivity  $\sigma_H = 1$  to identify the ground state, then the equations of motion for  $A_\alpha$  or  $C_\alpha$ :

$$\epsilon^{\alpha\beta\gamma}\partial_\beta C_\gamma = 0 \quad , \quad (4)$$

are devided into the equations of motion for  $C_m$

$$\frac{dj_m^{(e)}}{dt} = \lambda^{-1} \frac{dA_m}{dt} \quad , \quad (5)$$

and the constraint equation given by

$$\epsilon_{mn}\partial_m j_n^{(e)} = \lambda^{-1} B \quad , \quad B = -\epsilon_{mn}\partial_m A_n \quad , \quad (6)$$

with  $\epsilon_{mn} = -\epsilon_{nm} = 1$  and  $m, n = 1, 2$ .

These two groups of equations are the London's equations in 2 + 1-dimensions.

To see the relation of  $\sigma_H = 1$  condition with superconductivity let us recall that according to the definition  $\sigma_H := \frac{\sigma_0 \omega_c \tau}{1 + (\omega_c \tau)^2}$  [9] if the  $\sigma_H = 1$  in the quantum Hall-limit of IQHE where  $\omega_c \tau \gg 1$ , then the electric conductivity  $\sigma_0$  becomes equal to  $\omega_c \tau$  or  $\sigma_0 \gg 1$ . Followingly, the electric resistivity becomes very small indicating the superconductivity. It is intresting to mention that also in the classical Hall-limit, i. e. under the classical Hall-conditions or  $\omega_c \tau \ll 1$ , the  $\sigma_H = 1$  condition is related also with large electric conductivity  $\sigma_0 = (\omega_c \tau)^{-1}$  or with the superconductivity. This posibility is related with the interplay between the strength of the electric and magnetic fields and the mean free time or mobility or the temperature which is correlated with the relation between the high- $T_c$  superconductivity and the FQHE [11] where the mobility plays empirically an importent role. Furthermore, if  $\sigma_H$  in (3) becomes equal to any positiv fractional number indicating the FQHE situation,  $\sigma_0$  becomes according to the definition of  $\sigma_H$  again proportional to  $(\omega_c \tau)^{-1}$  which represents in view of  $\omega_c \tau \gg 1$  again the superconductivity [17]. Thus, for decreasing magnetic fields also the FQHE situation can result in the superconducting case if the preparation of the system allows [11]. Therefore : 1.) The usual superconductivity described by the standard London's equations should be related with the  $\sigma_H = 1$  case of IQHE with non-interacting

particles. 2.) The high- $T_c$  superconductivity should be related with the ground state of the non-interacting particles in FQHE *after* decreasing the magnetic fields [17].

It is important to mention that the empirical information from superconducting quantum interference devices (SQUID) support this point of view. According to Ginzburg [18] the diagram of the magnetic flux  $\Phi$  through the superconducting ring (with a weak link) with respect to the variation of the external magnetic flux has quasi plateaus on the integer magnetic flux  $\Phi = \phi_0, 2\phi_0, 3\phi_0, \dots$ . This diagram is similar to the diagram of Hall conductivity with respect to the filling factor in IQHE [9]. Roughly speaking, one could understand such a similarity, in view of the multiply connectedness of region in both cases and the essential role played by the pure gauge potential therein, if one relates the quantization of the Hall-conductivity  $\sigma_H = \nu \frac{e^2}{h}$  where  $\nu$  is the filling factor with the flux quantization according to  $B_{(SC)} \cdot S = \mathbf{Z} \frac{h}{e}$  in superconductivity (SC), where  $S$  is the area surrounded by the ring and  $\mathbf{Z}$  is an integer. The relation becomes obvious if one recalls that under QHE conditions  $\omega_c \tau \gg 1$  one has  $\sigma_H = \frac{ne}{B_{(QHE)}} = \frac{Ne}{B_{(QHE)} \cdot S'}$ , where  $B_{QHE} \gg B_{SC}$  and  $S' \ll S$  is the area of the ring itself where the charge carriers are. To be precise it has to be mentioned that  $S'$  is the area of a ring with a width of  $l_B$ , so that if  $S = \pi R^2$  is the empty area, then  $S' = 2\pi R \cdot l_B$  is the ring-area or the edge-area where the edge current should move around. Now, if  $B_{(QHE)} \cdot S' = B_{(SC)} \cdot S$ , then the two quantization in two different levels of external magnetic field strength are correlated by  $\nu = \frac{N}{\mathbf{Z}}$ . In this manner, where integer and fractional  $\nu$  implies IQHE and FQHE respectively, it is possible that QHE contains superconducting effect and vice versa. Moreover, one rediscovers the well known empirical fact about the proportion  $\frac{B_{(QHE)}}{B_{(SC)}} = \frac{S}{S'} = \frac{R}{2l_B}$  in the mentioned Corbino-type samples between the height of magnetic field in QHE and superconductivity [19]. This relation can be considered also as an explanation for the quantization of Hall-conductivity  $\sigma_H$  according to the flux quantization.

Such a relation between the QHE and Superconductivity should be realized by the Q-1-D superconducting systems [20]. These systems demonstrate under large magnetic fields along the low conductivity axis an IQHE-behaviour [20].

Theoretically, this behaviour becomes clear in view of the fact that the quantization of the 2 + 1-D field theories result in the 1 + 1-D quantum theories of "chiral" currents [22]. We showed, that the quantization of the 2 + 1-D IQHE model results in the 1-dimensional edge currents [12] which should



be described dynamically by  $1 + 1$ -D quantum theories [22]. Hence, there are the edge currents which demonstrate the IQHE or the quantization of the Hall-conductivity in ideal cases [15]. In other words, the IQHE-behaviour is described by a  $1 + 1$ -D quantum theory. On the other hand the mentioned Q-1-D superconductors should be described in our approach also as 1-dimensional quantum systems by  $1 + 1$ -D quantum theories. Thus, in view of the fact that these superconductors demonstrate the mentioned quantum  $1 + 1$ -behaviour which is equivalent to the edge current-behaviour, they might demonstrate also the IQHE-behaviour. Moreover, the example of Q-1-D superconductors shows that not only the FQHE [11] but also the IQHE is related with the superconductivity.

This considerations should be related with considerations in Ref. [21] where also the main theoretical object is a  $U(1)$  gauge potential which is introduced and gauged away frequently. It should be interesting to note here that, as in the case of Ref. [21], every decomposition of an electromagnetic gauge potential is equivalent to its mathematical definition according to its gauge transformation property  $A' = A + \partial\phi$  which contains the pure gauge potential  $\partial\phi$ .

On the other hand, the mentioned local equivalence of Ginzburg-Landau equations  $J_{(em)} = \Delta A$  with the definition of pure gauge potential discussed above should be understood so that obviously  $\Delta A = 0$  for pure gauge potentials. Furthermore, if we compare the strength of the increasing magnetic fields from weak and strong superconductivity to QHE which are described by London's, Ginzburg-Landau's and the Ohm's equations of QHE respectively, then we find that: Depending on the preparation of the particle system, with increasing of the exterior magnetic field the  $A$ ,  $\Delta A$  and  $dA$  of the pure gauge potential tends away from zero respectively but remain close to zero within the quantum mechanical uncertainty [2] [5]. The strength of exterior magnetic field here is a quantum mechanical measure of the non-vanishing of  $A$ ,  $\Delta A$  and  $dA = B$ , if one recall that the energy uncertainty is given by the  $\delta E = \frac{e\hbar B}{2m}$  [3]. This circumstance should be also understood according to the geometry on manifolds with different boundary structures [14]. Thus, depending on the preparation of the  $2 + 1$ -D particle system [1] [9] with increasing of the exterior magnetic field one has the empirical situations for which the London's-, the Ginzburg-Landau's- and the Ohm's equations of QHE are responsible respectively. Furthermore, recall that the generality of the flux quantization concept for all of these three cases and also the generality of the BCS-theory for the first two cases result from the generality of the pure gauge electromagnetic

potential which is according to our discussion the fundamental concept of all of them.

In conclusion let us mention that for arbitrary locally constant  $\sigma_H$  the variation of the action functional (3) with respect to  $A_\alpha$  results in the following equations which are the generalization of the equations (4) or (5)-(6) for arbitrary  $\sigma_H$ :

$$J_\alpha^{(em)} = \epsilon^{\alpha\beta\gamma} \sigma_H \partial_\beta A_\gamma \quad , \quad (7)$$

where we used  $J_\alpha^{(em)} = \lambda \epsilon_{\alpha\beta\gamma} \partial^\beta j_{(e)}^\gamma$  as given above according to the differential geometrical considerations about the current density involved in IQHE.

Therefore, if the  $\sigma_H$  becomes quantized in view of a proper preparation of the particle system for larger exterior magnetic fields [12] [15], then the spatial part of these equations become the Ohm's equations for the IQHE. Accordingly, if we perform a surface integral of the time component of (7) which is the constraint relation for constant magnetic field  $B$ , we obtain the well known defining relation for the quantum Hall-conductivity in the quantum limit  $\omega_c \tau \gg 1$  [9]:  $\sigma_H = \frac{ne}{B}$  which results in case of samples with proper prepared relations between  $B$  and  $n$  in quantized  $\sigma_H = \nu \frac{e^2}{h}$ .

This same results on IQHE including the Ohm's equations as equations of motion are obtained also from our Chern-Simons-Schrodinger-action for IQHE [12].

Footnotes and references

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- [2] In other words, one can transform the potential locally into the one equal to the gradient of the Phase of  $\psi$ . The global description of pure gauge character is given by the line integral of the potential which is equal to the mentioned phase. Furthermore, with pure gauge potential we mean everywhere a U(1)-type in multiply connected regions.
- [3] The uncertainty of  $B$  should be considered as a result of the energy uncertainty in Landau-levels i. e.  $E = (N + \frac{1}{2}) \frac{e\hbar B}{M_e}$ .
- [4] Herefor see S. Deser and R. Jackiw and S. Templeton, Phys. Rev. Lett., 48, 975 (1982)
- [5] See herefor [1d]. Furthermore, our model should be considered also in this category. Indeed the main concept in our 2 + 1-D model of superconductivity is the compensated or in other words the absent Coulomb potential which is represented in our model by the pure gauge potential within the quantum mechanical uncertainty. It is related with the so called arbitrary weak attraction after compensation of the Coulomb repulsion in the phononic approach which is responsible quantum mechanically for the Cooper pairs. It is also related with the energy uncertainty, the ground state energy and tunnel effects (see also the Josephson tunneling).
- [6] The concept of the 2-dimensional polarized phase space of the 2 + 1-D models is involved through the concepts of the Fermi-surface in the BCS-theory and in some other approaches in solid state physics. Recall that in view of the independency of the quantization from the polarization of the phase space also the Fermi-surface in momentum space can be the polarized phase space of the 2 + 1-D system. Furthermore, note also that for a Chern-Simons-action which is the typical action in 2 + 1- dimensions the momentum and position space are the same.

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